CHAPTER 2

INTRODUCTION TO TRACKING

The first step in the route to effective data analysis is the production of a track reconstruction procedure that minimizes systematic errors. The relevant data for tracking an event consists of the list of the MWPC wires supposedly "hit" by charged tracks. From these wires, a set of straight lines (tracks) representing charged particles passing through the detector must be found. In the absence of a magnetic field, tracks can be assumed to be straight-lines, with a possibly kink at the PbC, because interesting particles have a relatively low probability for scattering in the air or the chambers. The trajectory of a particle can thus be parameterized by the z-coordinate of its position using the set of equations

$$F_1 = x = x_0 + \frac{(x_1 - x_0)}{(z_1 - z_0)}(z - z_0)\,, \tag{2.1a}$$

$$F_2 = y = y_0 + \frac{(y_1 - y_0)}{(z_1 - z_0)}(z - z_0)\,, \tag{2.1b}$$

where the track is assumed to pass through the points (x_0, y_0, z_0) and (x_1, y_1, z_1). Since z_0 and z_1 can be chosen arbitrarily, each track is defined by four parameters; therefore, at least four[1] different MWPC's must be employed in the reconstruction of every track.

The "standard" method of track reconstruction with MWPC-like data begins by choosing four chambers as "crosshairs." Then, each possible combination of one wire from each crosshair chamber defines a candidate track. For each combination, the wires on the non-crosshair chambers are checked to determine whether the track candidate could represent an actual track, usually by requiring that a certain number of the non-crosshair chambers have active wires passing close to where the candidate track intersects them. This "combinatorial" method works; however, both the exponential growth in computation time with chamber occupancy and the inherent systematic errors have led to the search for an alternative algorithm. The systematics stem from explicit reliance on a given set (or sets) of crosshairs, thus incorporating the inefficiencies of these MWPC's directly into the tracker and thereby limiting its ability to detect real tracks.

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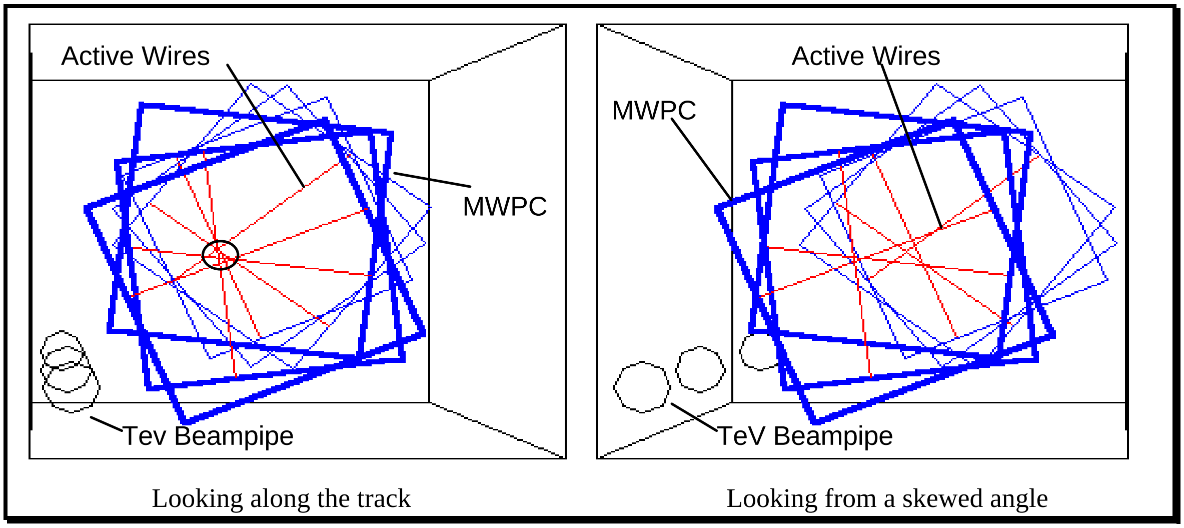
[1] More are required if several of the MWPC chambers included have wires oriented in the same direction.

2.1 Single-Vertex Hough Transform

Tracking “by eye” was the standard method of track reconstruction with bubble, cloud and streamer chambers. Pictures of the particle trails would be taken from various directions so as to obtain a three-dimensional view of the event. The pictures would then be visually scanned to find events of interest. With a three-dimensional computer display of the hit wires, one can rotate around the wires looking at two-dimensional projections.



Looking along the track

Looking from a skewed angle

Fig. 2-1: Projection of a Single Track

Some of these projections may be perpendicular to the trajectory of one or more tracks. In such a case, the hit wires will cluster around point of intersection with the projection; however, when looking from a skewed angle, the clustering is much less prominent—consisting of many fewer wires per cluster. This suggests that if the origin of the tracks is known, then the active wires can be projected onto some plane for which the maximum clustering will identify tracks. The natural implementation of this scheme is the *Single-Vertex Hough Transform*, henceforth referred to simply as the Hough Transform[2].

In order to Hough Transform an event from a given vertex (e.g. C0), one projects

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[2] Relevant references include:

D. H. Ballard and C. M. Brown, Computer Vision (Prentice-Hall, Inc.: 1982) Chapter 4.

H. Grote, “Pattern Recognition in High-Energy Physics,” Reports on Progress in Physics 50 (1987): 473-500.

P. V. C. Hough, "A Method and Means for Recognizing Complex Patterns", US Patent 3,069,654.

J. Illingworth and J. Kittler, “A Survey of the Hough Transform,” Computer Vision, Graphics, and Image Processing 44 (1988): 87-116.

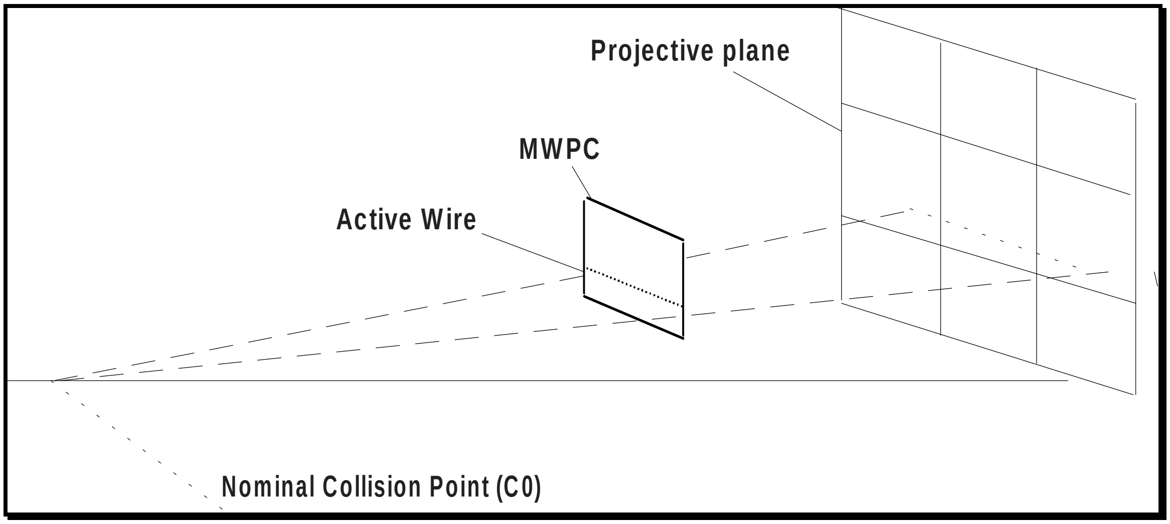

Fig. 2-2: Single-Vertex Hough Transform

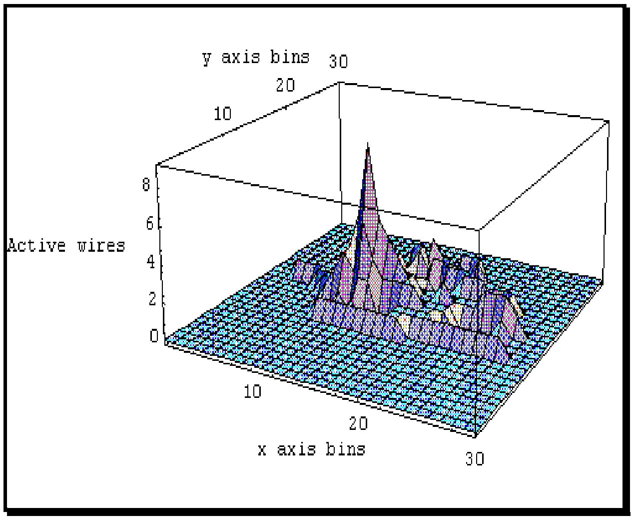

Fig. 2-3: Hough Transform of a Single Track

all active wires onto a two-dimensional grid as shown schematically in Fig. 2-2. One then counts the number of wires intersecting each square and looks for maxima. Peaks of height similar to the number of MWPC planes involved indicate the presence of a track originating near the assumed vertex and passing through the histogram square under consideration. An example where exactly one track present is shown in Fig. 2-3. The actual track parameters can be computed by performing a linear least-squares fit (LSF) of Eq. (2.1) through the contributing wires.

This method can easily be improved to account for chamber inefficiencies and the number of MWPC's that actually shadow (obscure the bin when viewed from the vertex) a given bin. Bins with excessive numbers of "hits" can be further subdivided to resolve track clusters into constituent tracks.

There are many potential difficulties in using scheme. First, tracks falling on or near the histogram bin boundaries can be lost. This can be remedied by using adaptive bin sizes or employing overlapping bins; however, if one is interested in finding tracks from several different vertices (the standard deviation in the z-distribution of collision vertices about C0 is on the order of 20"), then a Hough Transform must be done for each possible vertex. Second, if one is interested in finding tracks emanating from the PbC, the search must span a two-dimensional space of possible vertices. Clearly, in order to maximize the efficiency of this algorithm for finding actual tracks, a prohibitively large number of vertex locations must be scanned. Shifting the imaging point on the PbC by as little as a few millimeters can bring new tracks into focus because the detector planes are so close. Even if a resolution good enough resulted from a 0.5*cm x* 0.5*cm* mesh of positions, the algorithm, including all pattern-matching schemes, would have to be applied over one-thousand times to effectively cover the detector's acceptance at the PbC!

Clearly, a scheme independent of track origin is required which is able to resolve all tracks, irrespective of their trajectories, as long as they pass through enough MWPC's. One may notice that the combinatorial method fills this requirement; however, the Hough Transform can be generalized to also serve the purpose—without introducing the systematic errors inherent in preferred crosshair chambers.

2.2 Arbitrary-Vertex Hough Transform

A track can be generically defined as a tube passing through the detector and intersecting active wires on some minimum number of planes. The tube diameter depends on MWPC resolution, particle species, energy, etc. In Fig. 2-1, the intersection of such a tube with the projection forms the circular outline around the wire cluster. Of course, certain restrictions must be placed upon which tubes are admitted as tracks:

- At least N planes must have active wires intersecting the tube. N is a function of the total number of planes and perhaps other factors, such as the total number of active wires.
- A sufficient number of the N planes (at least 4) must have only one wire intersecting the tube to insure the tube contains only one track.
- A least-squares fit (LSF) of the wires intersecting the tube to a straight line should have a small enough χ_ν^2 to pass the criteria for a good fit[3].

These restrictions will be refined and others imposed as the discussion continues.

The algorithm now developed for searching out valid tubes will, henceforth, be referred to as the *Arbitrary-Vertex Hough Transform*—AVHT. To implement the AVHT,

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[3] See section 3.2 for a detailed exposition of the LSF and a development of the criteria for a good fit.

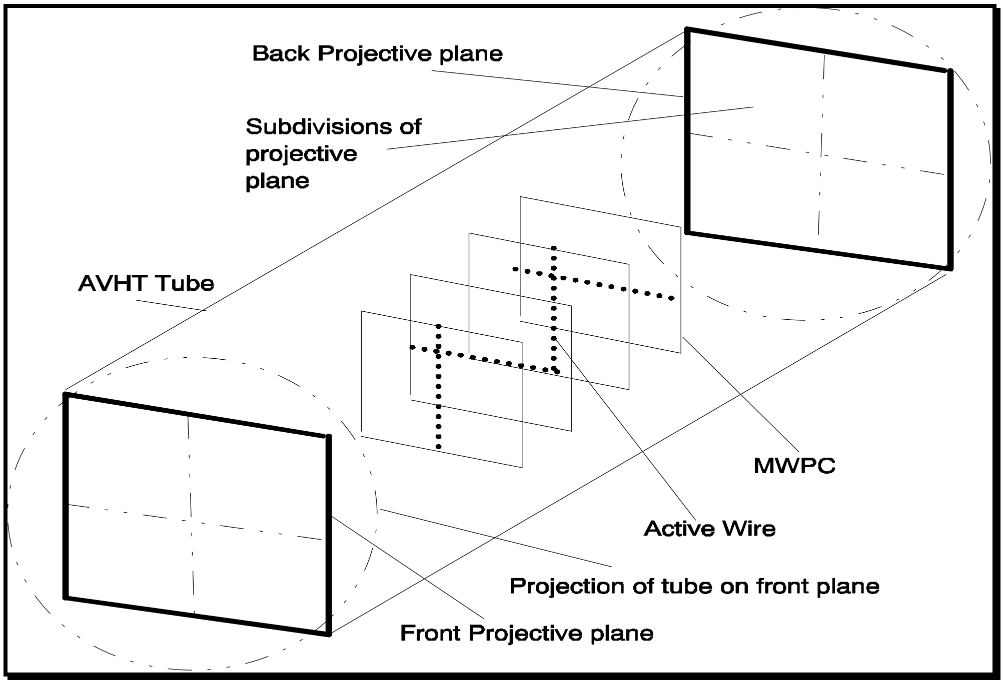

Fig. 2-4: Schematic First-Order AVHT Tube

a method for identifying candidate tubes similar to that for identifying candidate bins in the standard Hough Transform has been developed. Two "projective planes" (actually just equal-sized squares) are chosen such that the MWPC's of interest are located between them (Fig. 2-4). The size and positions of the first-order projective planes are chosen such that their area is minimized subject to the constraint that any track passing through enough of the MWPC's must intersect both of them or at least the circles circumscribed around them. The first-order AVHT tube consists of the "cylinder" containing the entire acceptance of the detector. It merely determines whether or not there are enough active wires in the whole event to constitute *any* tracks. With the definition of an AVHT tube as the "cylinder" circumscribing the current projective planes, a general recursive AVHT algorithm for finding tracks can be developed as in Fig. 2-5.

Parts of the AVHT algorithm will be examined in great detail in the following chapters along with an analysis of the various cuts applied. The heart of the AVHT method rests in what happens if a tube contains too much information—e.g. the number of tracks is indeterminate. In this case, the tube is divided into sixteen crossing and overlapping tubes, each with half the diameter of the current tube. For each of these sub-tubes, the algorithm of Fig. 2-5 is re-applied. Clearly, this will rapidly yield tubes of small radius clustering about lines of high wire density, exactly what is desired to effectively find tracks.

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A. Determine all the active wires intersecting the current AVHT tube.
B. Compute the number of tracks in the tube.
    1. Count the number, $p$, of MWPC planes that contribute active wires to the tube.
        a. If $p$ is less than MinPlanes then there are no tracks in the tube. Goto C.
    2. Compute the number, $n$, of possible tracks in the tube.
        ($n = \prod_{\substack{Contributing \\ Planes}} n_i$ where $n_i$ is the number of wires from the ith contributing plane.)
    3. If the number, $n$, of possible tracks is less than MaxCombs=16,
        a. Least-squares fit each possible track.
        b. Keep any track whose $\chi_v^2$ <= ChiCut.
        c. Goto C.
    4. Otherwise, if there are at least 5 planes contributing only 1 wire each,
        a. Least-squares fit the wires from these planes to get a "track."
        b. Require that this track satisfies $\chi_v^2$ <= ChiCut.
        c. For each plane contributing more than one wire,
            1. Compute which active wire comes closest to this track.
            2. If it is less than two wire spaces (0.2") away, add it to the track.
        d. If the number of planes contributing to the track >= MinPlanes
            1. Least-squares fit this track.
            2. If the track satisfies $\chi_v^2$ <= ChiCut
                a. Add it to list of tracks found.
                b. Goto C.
    5. Otherwise, the number of tracks is indeterminate
C. If the number of tracks is indeterminate, subdivide and check further
D. Everything about this tube is known.
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Fig. 2-5: AVHT Algorithm

The actual method of subdivision is fairly simple. As shown in Fig. 2-6, each projective plane is divided into quarters. There are sixteen combinations of one quarter from the front with one quarter from the back. Each of these combinations defines new projective planes and a new second-order AVHT tube. It is easy to see that any track passing through the parent tube must pass through at least one of these daughter tubes—so no information is lost in the subdivision. What does happen is that the average number of wires contributing to a tube decreases, unless the tube happens to enclose a track. Two important issues should be mentioned. First, the larger the first-order projective planes are, the more recursions it will take to reach a certain maximum resolution; so there is a computational incentive to minimize the area of these initial planes. Second, in order for the subdivision process not to lose information, the sub-tubes overlap so the same track might be identified in separate tubes. Care must be taken to differentiate these duplicate tracks from other real tracks.

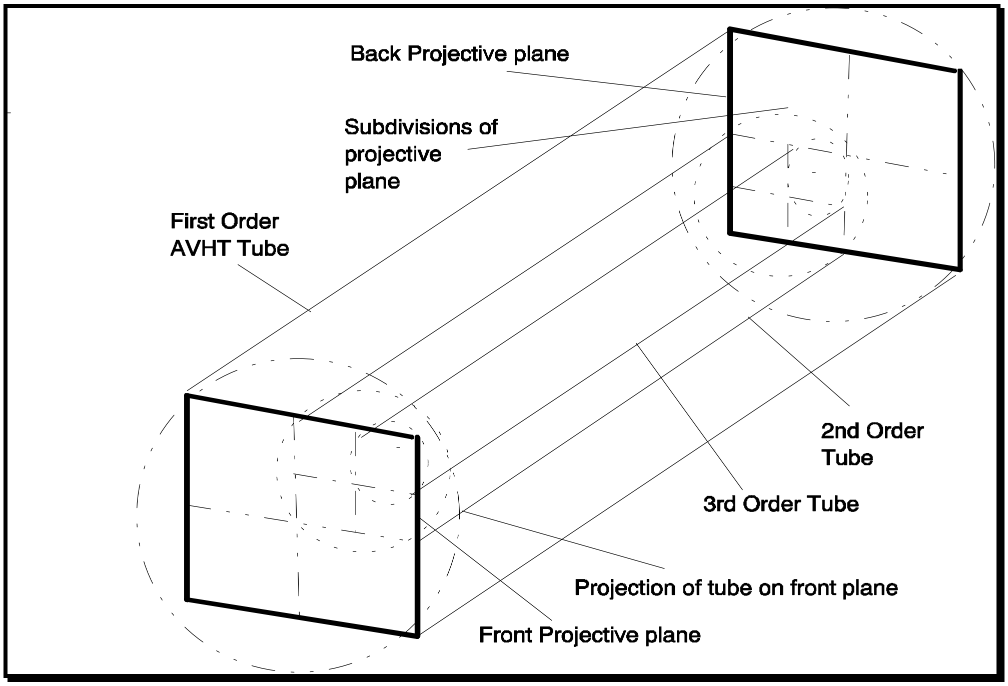

Fig. 2-6: AVHT Recursion Schematic

The determination of whether or not a given tube actually contains tracks is far from trivial. In fact, it is often the case that such a determination cannot be made, which requires the recursion just described in order to gain more specific information. However, there are several methods that quite effectively determine the presence of track(s) given sufficiently few wires in the tube. These consist of requiring that any track be constructed of wires from a large number of planes (*MinPlanes Cut*), using information from planes contributing only one active wire to guess which wires the other planes should contribute, and simply fitting all possible tracks, if there are few enough.

A major challenge for any tracking algorithm is the minimization of the occurrence of spurious tracks. Many methods for eliminating these will be discussed in the following chapters; however, it is suitable to discuss one of these methods here. Many spurious tracks consist mostly of wires from real tracks plus a few other active wires that happen to be in the right spot at the wrong time. Typically, these spurious tracks also do not have contributing wires from every plane; but, as will be seen in Section 3.2, setting *MinPlanes* low enough to find most real tracks allows many of these spurious tracks to be found as well. One method used to combat this is to apply the AVHT repeatedly to the same event, first with a *MinPlanes Cut* equal to the total number of planes under consideration and successively with looser and looser *MinPlanes Cuts*, until the actual *MinPlanes Cut* is reached. After each iteration, the wires contributing to tracks found are removed from the event so that they cannot assist in the formation of spurious tracks. While this scheme does have some disadvantage in that two real tracks can share active wires, the number shared will usually only be one or two unless the tracks are already “on top of” each other.

The AVHT method, as will be shown in succeeding chapters, can be reliably employed to find straight-line tracks through a given set of MWPC's. However, since T-864 places a sheet of lead in the middle of the detector to enable detection of $\gamma's$, the sets of MWPC's before and after the PbC must be treated separately. The results of each application must then be combined at the data analysis stage to determine what actually occurred.

The preceding overview of AVHT illustrates how it can be used to discover tracks independently of the occupancy or efficiency of any given chamber, although in the limits of extreme occupancy or inefficiency it cannot escape serious problems. Typically, in events where beam pipe conversions contribute significantly to the overall detector occupancy, only several adjacent chambers are affected. Because AVHT simultaneously examines all the chambers, it has a distinct advantage over the combinatorial method in that AVHT automatically uses wires on lower occupancy chambers to "see through" noise. It also accommodates chamber inefficiency by not depending on the response of any given chamber when determining the presence of a track. With the possession of an AVHT algorithm the details of implementation and determination of experiment-specific tracking parameters can now be explored. Then, an actual evaluation of efficiencies and systematic errors of the AVHT can be performed.